
RADICALS – ADVANCED

EXAMPLE 2: **Simplify each radical expression:**

$$A. \quad \sqrt{50xy^2} = \sqrt{5^2 \cdot 2 \cdot x \cdot y^2} = 5y\sqrt{2x}$$

$$B. \quad \sqrt{144a^{46}b^{19}} = \sqrt{12^2 \cdot (a^{23})^2 \cdot (b^9)^2 \cdot b} = 12a^{23}b^9\sqrt{b}$$

$$C. \quad \sqrt[3]{16x^{48}w^{14}} = \sqrt[3]{2^3 \cdot 2 \cdot (x^{16})^3 \cdot (w^4)^3 \cdot w^2} = 2x^{16}w^4\sqrt[3]{2w^2}$$

$$D. \quad \sqrt[4]{c^{20}d^{39}} = \sqrt[4]{(c^5)^4 (d^9)^4 d^3} = c^5 d^9 \sqrt[4]{d^3}$$

$$E. \quad \sqrt[5]{x^4 y^{53}} = \sqrt[5]{x^4 (y^{10})^5 y^3} = y^{10} \sqrt[5]{x^4 y^3}$$

$$F. \quad \sqrt[3]{54a^2 b^{27} c^{17}} = \sqrt[3]{3^3 \cdot 2 \cdot a^2 \cdot (b^9)^3 \cdot (c^5)^3 \cdot c^2} = 3b^9 c^5 \sqrt[3]{2a^2 c^2}$$

$$\begin{aligned} G. \quad \sqrt[5]{64x^3 y^5 z^{25} w^{47}} &= \sqrt[5]{2^5 \cdot 2 \cdot x^3 \cdot y^5 \cdot (z^5)^5 \cdot (w^9)^5 \cdot w^2} \\ &= 2yz^5 w^9 \sqrt[5]{2x^3 w^2} \end{aligned}$$

As we'll see in the following example, fractional exponents are indispensable for some radical problems.

EXAMPLE 3: **Simplify each radical expression:**

$$A. \quad \sqrt{\sqrt[3]{x}} = \left(x^{1/3}\right)^{1/2} = x^{1/6} = \sqrt[6]{x}$$

$$B. \quad \sqrt[4]{\sqrt[3]{\sqrt{2}}} = \left[\left(2^{1/2}\right)^{1/3}\right]^{1/4} = 2^{1/24} = \sqrt[24]{2}$$

$$c. \quad \sqrt[12]{a^9} = \left(a^9\right)^{1/12} = a^{9/12} = a^{3/4} = \sqrt[4]{a^3}$$

NEW Problems:

$$\text{Single Radical:} \quad \sqrt{x} \sqrt[3]{y} \quad \sqrt[3]{ab^2} \sqrt[5]{a^2b}$$

$$\text{Single Term:} \quad a\sqrt{x} + \sqrt{x^3} \quad [\text{Assume } x \geq 0]$$

Homework

1. Simplify: a. $\sqrt{200xy^2z^3w^{16}v^{37}}$ b. $\sqrt{72a^{25}b^{12}}$
2. Simplify: a. $\sqrt[3]{16ab^2c^3d^{27}e^{50}}$ b. $\sqrt[3]{54x^{22}y^{32}}$
3. Simplify: a. $\sqrt[4]{32x^4y^{16}z^{39}}$ b. $\sqrt[4]{81a^4b^{196}}$
4. Simplify: a. $\sqrt[5]{64u^{32}v^{54}}$ b. $\sqrt[5]{486w^4z^{32}}$
5. Use your calculator to verify Example 3B.
6. DISprove the conjecture: $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$ [Hint: use numbers]
7. Simplify each nested radical:
 - a. $\sqrt[4]{\sqrt{7}}$ b. $\sqrt[3]{\sqrt{10}}$ c. $\sqrt[5]{\sqrt[7]{x}}$
 - d. $\sqrt[4]{\sqrt[4]{n}}$ e. $\sqrt[5]{\sqrt[3]{\sqrt{a}}}$ f. $\sqrt[4]{\sqrt[3]{\sqrt{x}}}$

8. Simplify:

a. $\sqrt[6]{x^2}$

b. $\sqrt[8]{n^6}$

c. $\sqrt[10]{x^5}$

d. $\sqrt[9]{z^3}$

e. $\sqrt[20]{a^{15}}$

f. $\sqrt[21]{n^{14}}$

g. $\sqrt[30]{m^{26}}$

h. $\sqrt[88]{x^{77}}$

9. Simplify $\sqrt{\sqrt[3]{4\sqrt{3}}}$. Use your calculator to verify your answer.

Solutions

1. a. $10yzw^8v^{18}\sqrt{2xzv}$

b. $6a^{12}b^6\sqrt{2a}$

2. a. $2cd^9e^{16}\sqrt[3]{2ab^2e^2}$

b. $3x^7y^{10}\sqrt[3]{2xy^2}$

3. a. $2xy^4z^9\sqrt[4]{2z^3}$

b. $3ab^{49}$

4. a. $2u^6v^{10}\sqrt[5]{2u^2v^4}$

b. $3z^6\sqrt[5]{2w^4z^2}$

5. Does your verification prove that the answer is right?

6. For instance, $\sqrt{9+16} = \sqrt{25} = 5$, whereas $\sqrt{9} + \sqrt{16} = 3 + 4 = 7$. Note that these numbers constitute a *counterexample*.

7. a. $\left(7^{1/4}\right)^{1/2} = 7^{1/8} = \sqrt[8]{7}$

b. $\sqrt[6]{10}$

c. $\left(x^{1/7}\right)^{1/5} = x^{1/35} = \sqrt[35]{x}$

d. $\sqrt[16]{n}$

$$\text{e. } \left(\left(a^{1/3} \right)^{1/2} \right)^{1/5} = a^{\frac{1}{3} \cdot \frac{1}{2} \cdot \frac{1}{5}} = a^{1/30} = \sqrt[30]{a} \qquad \text{f. } \sqrt[24]{x}$$

$$\begin{array}{ll} \text{8. a. } \sqrt[6]{x^2} = x^{2/6} = x^{1/3} = \sqrt[3]{x} & \text{b. } \sqrt[8]{n^6} = n^{6/8} = n^{3/4} = \sqrt[4]{n^3} \\ \text{c. } \sqrt{x} & \text{d. } \sqrt[3]{z} \quad \text{e. } \sqrt[4]{a^3} \quad \text{f. } \sqrt[3]{n^2} \quad \text{g. } \sqrt[15]{m^{13}} \quad \text{h. } \sqrt[8]{x^7} \end{array}$$

$$\text{9. a. } \sqrt[24]{3}; \text{ the problem and the answer are each about } 1.046839.$$